Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/41**

Paper 4 Mechanics **October/November 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.





**[Turn over**

**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only

ISW Ignore Subsequent Working

SOI Seen Or Implied

SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\pm 1.6g \times x \times \frac{3}{5}$ [=9.6$x$] or $\pm \frac{1}{2} \times 1.6 \times 20^2$ [= 320] | **B1** | For either the correct potential energy or kinetic energy term. Need not be evaluated. |
| | $\frac{1}{2} \times 1.6 \times 20^2 = 1.6g \times x \sin\alpha$ where $\sin\alpha = \frac{3}{5}$ | **M1** | Attempt at energy equation; 2 relevant terms. Dimensionally correct but allow sign errors. Allow sin/cos mix and sin(36.869…) but $\sin\alpha$ (oe) must have been substituted. M0 for $1.6g \times x \times \frac{3}{4}$. |
| | $x = \frac{100}{3}$ | **A1** | Allow 33.3. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Attempt to resolve horizontally or vertically to form an equation. | ***M1** | Correct number of terms; allow sin/cos mix; allow sign errors – do not award this mark if using $T$ for both (see **SC** later). |
| | $T_1 \cos 35 = T_2 \cos 40$ | **A1** | Must be different $T$s. |
| | $T_1 \sin 35 + T_2 \sin 40 = 2.4g$ | **A1** | If same $T$s, then **SC B2** only for this equation. |
| | Attempt to solve for either tension. | **DM1** | From equations with correct number of relevant terms. Must get a value for at least one tension. E.g. $T_2\left(\frac{\cos 40}{\cos 35} \times \sin 35 + \sin 40\right) = 24$ |
| | $T_1 = 20.4$ N and $T_2 = 19.0$ N | **A1** | $T_1 = 19.033621\ldots$ $T_2 = 20.353166\ldots$ awrt 20.4 for $T_1$ www and 19(.0) for $T_2$. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Distance = 50.4 m | **B1** | Allow $\frac{252}{5}$. |
| | | **1** | |
| 3(b) | $v_1 = 12.6 - (62-48)a$ | **M1** | Use of suvat for first section of deceleration. $12.6 \pm (62-48)a$ only. |
| | $0 = v_1 - 2a \times (70-62)$ | **M1** | Use of suvat for second section of deceleration. An expression for the velocity at 62 seconds must be $\pm 2a \times (70-62)$. |
| | $a = 0.42$ | **A1** | –0.42 scores A0. |
| | | **3** | |
| 3(c) | Speed at time $t = 62$ is $6.72\,\text{m s}^{-1}$ | **B1** | This may be seen in part **(b)** but must be used in part **(c)** to get this mark. |
| | $s_2 = (48-8) \times 12.6$ $[= 504]$<br>$s_3 = 0.5 \times (12.6 + \textit{their}\ 6.72) \times (62-48)$ $\left[= 135.24 \text{ or } \frac{3381}{25} \text{ oe}\right]$<br>or $\textit{their}\ 6.72 \times (62-48) + 0.5 \times (62-48) \times (12.6 - \textit{their}\ 6.72)$<br>$s_4 = 0.5 \times \textit{their}\ 6.72 \times (70-62)$ $\left[= 26.88 \text{ or } \frac{672}{25} \text{ oe}\right]$ | **B2FT** | B2 FT for any 2 correct, B1 FT for any 1 correct – follow through *their* value of $v_1$ where $0 < v_1 < 12.6$ but must have come from the correct equations seen in part **(b)**. Allow correct value of $v_1$ from $a = -0.42$ where $v_1 = 12.6 + (62-48)a$ and $v_1 = -2a \times (70-62)$. |
| | Average speed $= 10.236\,\text{m s}^{-1}$ | **B1** | Allow 10.2 or better oe e.g. $\frac{2559}{250}$, $10\frac{59}{250}$. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $-0.4\times6g=6a$ | ***B1** | Resolve horizontally using Newton's second law; 2 relevant terms; must be either $-0.4\times6g=6a$ or $0.4\times6g=6a$. |
| | $v^2=20^2+2\times(-4)\times12.5$ | **DM1** | Use complete suvat method to get an equation in $v$ or $v^2$ – must be using $u = 20$, $s = 12.5$ and *their a*. |
| | $v^2=300 \Rightarrow v=10\sqrt{3}$ | **A1** | AG. Condone correct expression for $v$ or $v^2$ followed by correct answer. |
| | **Alternative method for Question 4(a)** | | |
| | $RF=0.4\times6g$ | ***B1** | Correct application of $F=\mu R$ for $P$. |
| | $0.5\times6\times20^2-0.5\times6\times v^2=12.5\times(0.4\times6g)$ | **DM1** | 3 relevant terms; dimensionally correct; allow sign errors only. |
| | $v^2=300 \Rightarrow v=10\sqrt{3}$ | **A1** | AG. Condone correct expression for $v$ or $v^2$ followed by correct answer. |
| | | **3** | |
| 4(b) | $6\times10\sqrt{3}=(6+2)v'$ | **M1** | For use of conservation of momentum, 3 non-zero terms, allow sign errors. Use of 20 is M0. |
| | $v'=7.5\sqrt{3}$ | **A1** | 12.99038… |
| | Initial KE $=\frac{1}{2}\times6\times\left(10\sqrt{3}\right)^2[=900]$<br>Final KE $=\frac{1}{2}\times8\times\left(7.5\sqrt{3}\right)^2[=675]$ | **B1** | Either initial kinetic energy or final kinetic energy correct. Allow unsimplified. |
| | Loss of KE = 225 J | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(c) | $0=\left(their\,7.5\sqrt{3}\right)^2+2\times\left(their-4\right)\times s$ | **M1** | Use complete suvat method to find distance. This must be using *their* $v'$ from part **(b)**, so it is dependent on scoring the first M mark in part **(b)** and either *their a* from part **(a)**, or from $\pm 0.4\times 8g=8a$. |
| | [Distance =] 21.1 m | **A1** | 21.1 or better (21.09375). |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | Resolving parallel to the slope at *A* or *B* to form an equation. | ***M1** | Correct number of terms; allow sign errors; allow sin/cos mix. |
| | $1.6g\sin 50 - T - F_B = 0$ | **A1** | If using the same *F*s, then M1A1A0B1 max. |
| | $T - F_A - 1.2g\sin 40 = 0$ | **A1** | System equation (must be four different terms): $1.6g\sin 50 - F_B - F_A - 1.2g\sin 40 = 0$ only scores M1A1A1. Any sign errors scores M1 only. |
| | $R_A = 1.2g\cos 40$ or $R_B = 1.6g\cos 50$ | ***B1** | Either correct. Must be explicitly linked to the correct contact (so could be seen on a diagram), or as part of a resolving parallel to the slope equation(s) (so must be combined with $\mu$). |
| | $F_A = 1.2g\mu\cos 40$ or $F_B = 1.6g\mu\cos 50$ | ***M1** | Use of $F = \mu R$ at either *A* or *B*. Must be explicitly linked to the correct contact (could be seen on a diagram) or as part of a resolving parallel to the slope equation(s). Allow sin/cos mix error only. |
| | $1.6g\sin 50 - 1.6g\mu\cos 50 = 1.2g\sin 40 + 1.2g\mu\cos 40$ | **DM1** | Eliminating *T*, $F_A$ and $F_B$ to form an equation in $\mu$ only. |
| | $\left[\mu = \dfrac{1.6g\sin 50 - 1.2g\sin 40}{1.2g\cos 40 + 1.6g\cos 50} \Rightarrow\right] \mu = 0.233$ | **A1** | 0.23326119… |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a)(i) | Power $=19.5\,\text{kW}$ | **B2** | Or B1 for either $650\times30$ or 19 500. |
| | | **2** | |
| 6(a)(ii) | *their* $19500+9000=DF\times30$ | **B1FT** | Oe FT *their* 19.5 in watts only. |
| | $DF-650=1300a$ | **M1** | Newton's second law horizontally; 3 relevant terms; dimensionally correct but allow sign errors; allow with *their* driving force or just *DF*. |
| | $a=\frac{3}{13}=0.231\,\text{m s}^{-2}$ | **A1** | 0.23076923… |
| | **Alternative scheme for 6(a)(ii)** | | |
| | $9000=DF\times30$ | ***B1** | oe e.g. $DF=\frac{9000}{30}$. |
| | $DF=1300a$ | **DM1** | Resolving horizontally using Newton's second law; 2 relevant terms; dimensionally correct but allow sign errors. |
| | $a=\frac{3}{13}=0.231\,\text{m s}^{-2}$ | **A1** | 0.23076923… |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $DF = \dfrac{11500}{v}$ | **B1** | oe e.g. $DF \times v = 11500$. |
| | Attempt at Newton's second law. | **M1** | 4 relevant terms, *their DF* or just *DF*; allow sign errors: allow sin/cos mix; allow $g$ missing. |
| | $\dfrac{11500}{v} + 1300 \times g \times 0.08 - (1000 + 20v) = 0$ | **A1** | Correct equation. |
| | Speed = $25\,\text{m}\,\text{s}^{-1}$ | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7 | $30.6-0.9\times 8=\frac{1600}{8^2}+8k$ | ***M1** | Use velocity at $t=8$ to set up a linear equation in $k$ only. Allow a slip in one value or sign only. |
| | $k=-0.2$ | **A1** | |
| | $\frac{1600}{t^2}+(their\,k)\times t=0 \Rightarrow t=\ldots$ | **DM1** | Attempt to find the value of $t$ when the particle comes to rest using the correct expression for $v$, set equal to zero with *their* negative value of $k$. Must find a positive value for $t$ (for reference, $t = 20$). |
| | Attempt to integrate $v$ for one of the 3 intervals | ***M1** | Increase power by 1 and a change in coefficient in at least one term (which must be the same term); $s = vt$ is M0. |
| | $s=\frac{7.2}{3}t^3(+c)$ | **A1** | May be unsimplified (for reference, limits are from 0 to 2). |
| | $s=30.6t-\frac{0.9}{2}t^2(+c)$ | **A1** | May be unsimplified (for reference, limits are from 2 to 8). |
| | $s=\frac{1600}{-1}t^{-1}+\frac{k}{2}t^2(+c)$ | **A1FT** | May be unsimplified (for reference limits are from 8 to 20). Follow through *their* value of $k$ or just $k$ only. |
| | Either 19.2 or 156.6 or $\pm 86.4$ | **B1** | One correct distance found. Allow unsimplified e.g. $(216-59.4)$ or $\frac{1}{2}\times(8-2)\times(28.8+23.4)$ etc. |
| | Distance $=19.2+(216-59.4)+(-120-(-206.4))=262.2$ m | **B1** | This mark can be awarded if no integration is shown oe. e.g. $\frac{1311}{5}$. Condone 262 www. |
| | | **9** | |